Yale University Department of Mathematics
Math 604 Introduction to Quadratic Forms
Fall Semester 2016

Instructor:	Asher Auel	**Lecture:**	LOM 205
Office:	LOM 210	**Time:**	Fri 1:30 – 3:45 pm
Web-site:	http://math.yale.edu/~auel/courses/604f16/		

Introduction: This course will provide an introduction to the algebraic theory of quadratic forms over fields. Topics will include the elementary invariants (discriminant and Hasse invariant), classification over various fields (real numbers, rational numbers, p-adic numbers, finite fields, and rational function fields), isotropy and the u-invariant, the Witt group, and Pfister forms. Along the way, we will also cover the basic theory of quaternion algebras. Special topics might include local-global principals (e.g., Hasse-Minkowski), sums of squares (e.g., Lagranges theorem and Pfisters bound for the Pythagoras number), Milnor K-theory, and orthogonal groups, depending on the interests of the participants.

Grading: Your final grade will be based on class participation, monthly problem sets, and a final project.

Prerequisites: Some prior experience with linear algebra and Galois theory will be necessary.

Topics covered: Subject to change.

(1) Background material. Square classes. Quadratic forms and symmetric bilinear forms. Diagonalization of quadratic forms. Binary and ternary forms. Norm form for quadratic extension.

(2) Elementary invariants. Dimension. Discriminant and signed discriminant. Signature over ordered fields. Sylvester's law of inertia. Hasse-Witt invariant and Clifford invariant. Norm form for quaternion algebra.

(3) Quaternion algebras. Symbols. Beginnings of Milnor K-theory. Central simple algebras and the Brauer group. Clifford algebras.

(4) Witt group and Witt equivalence. Hyperbolic forms and isotropy. Cancellation.

(5) Fundamental filtration of the Witt group. Pfister forms. Multiplicative forms.

(6) Classification over the real numbers. Classification over finite fields. Classification over p-adic numbers. Hasse-Minkowski and classification over the rational numbers.

(7) Anisotropic forms and the u-invariant of a field.

(8) (Optional.) Sums of squares and the Pythagoras numbers. Pfister's bounds and recent results.

(9) (Optional.) Integral quadratic forms and lattices. Lagrange's theorem. Theta functions.

(10) (Optional.) Galois cohomology. Milnor conjectures.

(11) (Optional.) Orthogonal groups. Generators for the orthogonal group. Eichler transvections.