YALE UNIVERSITY DEPARTMENT OF MATHEMATICS
MATH 373/573 ALGEBRAIC NUMBER THEORY
SPRING SEMESTER 2019

Instructor:	Professor Asher Auel	**Lecture:**	LOM 214
Office:	LOM 210	**Time:**	Tue Thu 9:00 - 10:15 am
Text:	*Algebraic Number Theory (v3.07)*, J. S. Milne Available at http://www.jmilne.org/math/		
Web-site:	http://gauss.math.yale.edu/~auel/courses/373s19/		

Introduction: The main object of study in Algebraic Number Theory are number fields (finite extensions of the field of rational numbers) and their rings of algebraic integers (those elements that are roots of monic polynomials with integer coefficients). Rings of algebraic integers have a theory of prime ideals that encode many interesting properties about the usual rational prime numbers, for example, the decomposition of primes in the Gaussian integers detects whether they can be written as a sum of two squares (like 5) or not (like 7). One of the most fundamental algebaic invariants of a number field is its ideal class group, which measures, among other things, the failure of unique factorization in the ring of integers. One of the fundamental theorems in Algebraic Number Theory is the finiteness of the ideal class group. Another is about the structure of units in the ring of integers. Both of these fundamental invariants are also mysteriously hidden in the Dedekind zeta function of a number field, analogous to the classical Riemann zeta function.

Grading: Your final grade will be calculated according to the table at right. Notice that equal overall emphasis is placed on exams and on homework assignments.

Homework	50 %
Midterm	20 %
Final Exam	30 %

Exams: The midterm exam will take place in-class on Thursday 7 March. The final exam will take place during final exam period. Make-up exams will only be allowed with a dean's excuse.

Homework: There will be problems sets assigned every week or two. The problem sets will be posted on the course web-site syllabus page. Your lowest problem set score will be dropped from your final grade calculation.

Consider (as you would for any other class) the pieces of paper you turn in as a final copy: written neatly and straight across the page, on clean paper, stapled together, with nice margins, lots of space, and well organized. If you haven't already, you might consider taking the opportunity to learn LaTeX.

Group work, honestly: Working with other people on mathematics is not only allowable, but is highly encouraged and fun. You may work with anyone (e.g., other students in the course, students not in the course, tutors) on the rough draft of your problem sets. If done right, you'll learn the material better and more efficiently working in groups. The golden rule is:

> You may work with anyone on *solving* your homework problems,
> but you must *write* up your final draft by yourself.

Writing up the final draft is as important a process as figuring out the problems on scratch paper with your friends. Mathematical writing is very idiosyncratic—it is easy to tell if papers have been copied from others or from the internet—just don't do it! You will not learn, and will engage in academic dishonesty, by copying solutions! Also, if you work with people on a particular assignment, you *must list your collaborators at the top of the paper*, as well as any resources (e.g., Wikipedia) used beside the text book. All claims in your solutions must be fully and rigorously justified. You are free to cite results from lecture, from the book, and within reason, from anywhere else (as long as it doesn't make the problem trivial). Make the process fun, transparent, and honest.

Prerequisites: Previous exposure to abstract algebra, field theory, and Galois theory is absolutely required; some experience with complex analysis might be helpful for special topics. For example, the contents of Math 350 Introduction to Abstract Algebra, Math 370 Fields and Galois Theory, and Math 310 Introduction to Complex Analysis, would suffice. Math 380 Modern Algebra is recommended, but only required for graduate students.

Topics covered: Subject to change.

(1) Rings of algebraic numbers, integral closure.
(2) Discriminant and different. Power bases. Algorithms for computing the ring of integers.
(3) Prime ideals, decomposition of ideals into products of prime ideals, splitting of primes.
(4) Discrete valuation rings and their extensions, ramification theory. Completions and p-adic fields.
(5) Ideal class group and finiteness.
(6) Dedekind unit theorem.
(7) (Optional topic) Dedekind zeta function and special values.
(8) (Optional) Ring of adeles and group of ideles, Fourier analysis.