Yale University Department of Mathematics
Math 350 Introduction to Abstract Algebra
Fall 2016

Problem Set # 5 (due at the beginning of class on Friday 14 October)

Reading: DF 3.4, 4.1–4.3.

Problems:

1. DF 3.4 Exercises 2, 4*, 7, 8*.

2. DF 3.5 Exercises 10.

3. DF 4.1 Exercises 3*, 4, 6, 7*, 8, 9.

4. DF 4.2 Exercises 2, 10*, 11, 14.

5. DF 4.3 Exercises 3, 5*, 25, 29*, 31, 32, 34.

6. *Finite vector spaces.* Let V be an $\mathbb{F}_p$-vector space of (finite) dimension n.

(a) What is the isomorphism type of the underlying finite abelian group $(V, +)$?

(b) Show that the automorphism group $\mathrm{Aut}((V, +))$ of the abelian group $(V, +)$ is isomorphic to the group $\mathrm{GL}(V)$ of $\mathbb{F}_p$-linear vector space isomorphisms $\varphi : V \to V$ and that this group is also isomorphic to $\mathrm{GL}_n(\mathbb{F}_p)$.

(c) Compute the order of $\mathrm{Aut}(\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z})$. Find an automorphism of order 7.

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