Yale University Department of Mathematics
Math 350 Introduction to Abstract Algebra
Fall 2016

Problem Set # 2 (due at the beginning of class on Friday 23 September)

Notation: Z_n is an abstract cyclic group written multiplicatively.

Reading: DF 1.4, 1.7, 2.1–2.3.

Problems:

1. DF 1.7 Exercises 5, 17 (this gives another proof of 1.1 Exercise 22), 18*, 19*.

2. DF 2.1 Exercises 2, 6*, 7, 8, 9*, 14, 15.

3. DF 2.3 Exercises 2, 5, 8*, 10, 12*, 20, 21*, 22*, 23* (Hint: What does 22 tell you about the order of 5 in $(\mathbb{Z}/2^n\mathbb{Z})^\times$?), 25, 26*.

4. DF 2.4 Exercises 6, 7, 8, 9*, 11* (Hint: What are the orders of elements in S_4?), 13, 12*, 14*, 15, 19.

5. Let $\mathbb{F}_4 = \{0, 1, x, y\}$. Prove that there are operations $+$ and $\cdot$ on $\mathbb{F}_4$, such that $1 + x = y$ and $x^2 = y$, making $\mathbb{F}_4$ into a field. Note that the four elements of $\mathbb{F}_4$ are distinct! Essentially the problem is to fill out the addition and multiplication tables:

$+$	0	1	x	y
0				
1				
x				
y				

$\cdot$	0	1	x	y
0				
1				
x				
y				

You already know certain rows and columns by properties of 0 and 1 in a field!

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