Yale University Department of Mathematics
Math 225 Linear Algebra and Matrix Theory
Spring 2018

Problem Set # 11 (due in class Thursday April 26)

Notation: Let V be an inner product space and $S \subset V$ a nonempty subset. Define the **orthogonal complement** $S^\perp = \{x \in V \ : \ \langle x, y \rangle = 0 \text{ for all } y \in S\}$ to be the set of all vectors in V orthogonal to every vector in S. It's immediate from the properties of an inner product that $\{0\}^\perp = V$ and $V^\perp = \{0\}$.

An important result (which is a consequence of Theorem 6.6 that you will prove below) is that if $W \subset V$ is a finite dimensional subspace, then $V = W \oplus W^\perp$. Furthermore, if $\{v_1, \ldots, v_k\}$ is an orthonormal basis of W, then the linear map $T : V \to W$ defined by $T(y) = \sum_{i=1}^k \langle y, v_i \rangle v_i$ is called the **orthogonal projection** to W.

Recall that if $A \in \mathsf{M}_{n\times n}(\mathbb{C})$ then A^* is the conjugate transpose matrix.

Reading: FIS 6.1–6.4.

Problems:

1. FIS 6.1 Exercise 23abc (Here F^n has the standard inner product! Hint for part (c): prove that $\langle Qx, Qy \rangle = \langle x, y \rangle$ first, then use the previous parts. To do this, expand in the standard basis.)

2. FIS 6.2 Exercises 1 (If true, then either cite or prove it, if false then provide a counterexample), 2hi (in part h, use the Frobenius inner product), 11, 12, 13, 15 (in part b, recall that $\phi_\beta : V \to F^n$ is the map $\phi_\beta(x) = [x]_\beta$), 18.

Think about, but do not hand in: 7, 8, 19, 20, 21.

3. FIS 6.3 Exercises 1 (If true, then either cite or prove it, if false then provide a counterexample), 2ac (for part a, use the standard inner product on $\mathbb{R}^3$), 10 (you can take FIS 6.1 #20 for granted), 12, 18.

Think about, but do not hand in: 2, 3, 8, 10, 14, 15.

4. FIS 6.4 Exercises 2ad, 9.

Think about, but do not hand in: 4, 6, 11, 13, 17, 20.

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