

Problem Set # 8 (due 4 pm Thursday 26 March 2014)

Notation: Let $g(t)$ be a polynomial with coefficients in a field F . Let $T : V \rightarrow V$ be a linear operator on an F -vector space V . Then we define a linear operator $g(T) : V \rightarrow V$ as follows. First, for an integer $n \geq 1$, denote by $T^n : V \rightarrow V$ the n -fold composition

$$T^n = \underbrace{T \circ T \circ \cdots \circ T}_{n \text{ times}}$$

of T with itself. We define $T^0 = I_V$. Writing

$$g(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0,$$

then we define g applies to the linear operator T as

$$g(T) = a_n T^n + a_{n-1} T^{n-1} + \cdots + a_1 T + a_0 I_V,$$

where the scalar multiples and sums are being taking inside the F -vector space $\mathcal{L}(V)$ of linear operators on V .

For an $n \times n$ matrix A with coefficients in F , there is an analogous definition of $g(A)$. Make sure you understand the content of the equality of linear operators $g(L_A) = L_{g(A)}$ on F^n .

Reading: FIS 5.1–5.2

Problems:

1. FIS 5.1 Exercises 1 (If true, then either cite or prove it, if false then provide a counterexample), 2bd, 3bd, 4de, 8, 9, 10, 11, 14, 17, 23.

Think about, but do not hand in: 15, 16, 19, 20.

2. FIS 5.2 Exercises 9, 10, 11 (Hint: The easiest way to do part (a) is to do FIS 5.1 Exercise 16, which in turn means doing FIS 2.3 Exercise 13).