Yale University Department of Mathematics
Math 225 Linear Algebra and Matrix Theory
Spring 2014

Problem Set # 2 (due 4 pm Wednesday 29 January 2014)

Notation: Let F be a field, 0_F its (unique) additive identity, and 1_F its unique multiplicative identity. There is a natural map $\mathbb{Z} \to F$ so that for each integer $n \in \mathbb{Z}$, we can consider $n \in F$ as follows: if $n = 0$, then map n to 0_F; if $n > 0$, then map n to the sum $1_F + \cdots + 1_F$ of 1_F with itself taken n times; if $n < 0$, then map n to the sum $(-1_F) + \cdots + (-1_F)$ of -1_F with itself taken $-n$ times.

We say that the field F has **characteristic zero** if the map $\mathbb{Z} \to F$ is injective. However, it can happen that this map is not injective. For a prime number p, we say that F has **characteristic** p if this map sends p to 0_F. For example, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ have characteristic zero, while $\mathbb{F}_p$ has characteristic p. It is a theorem from abstract algebra that either a field has characteristic zero or the smallest positive integer mapping to 0_F is actually a prime number.

Reading: FIS 1.4–1.6

Problems:

1. FIS 1.4 Exercises 1 (Either cite, prove, or provide a counterexample), 3bc (Show your work), 5eh (Show your work), 11, 13, 15.

2. FIS 1.5 Exercises 1 (Either cite, prove, or provide a counterexample), 2bcde, 3 (Just stare at it, you do not need to show your work), 8 (In part a, the book writes R for $\mathbb{R}$), 9, 10, 11 (The book writes Z_2 for $\mathbb{F}_2$), 12, 18, 20.

3. Let F be a field. Prove that two vectors (a, b) and (c, d) in F^2 are linearly dependent if $ad-bc = 0$ and are linearly independent if $ad - bc \neq 0$.

Yale University, Department of Mathematics, 10 Hillhouse Ave, New Haven, CT 06511
E-mail address: asher.auel@yale.edu