Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Spring 2012

Problem Set # 4 (due Friday 17 February 2012)

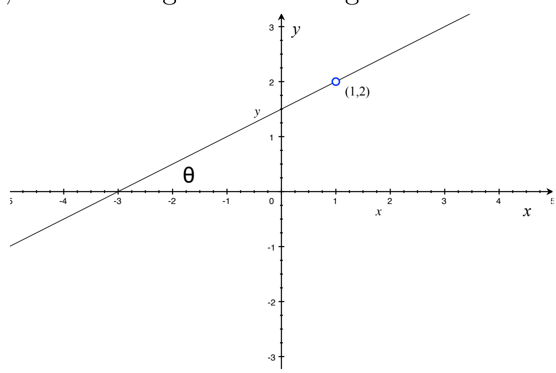
Lift: If $\gamma : \mathbb{R} \to \mathbb{R}^2$ is a parameterized curve in the x-y-plane given by $\gamma(t) = (\gamma_1(t), \gamma_2(t))$, and $f : \mathbb{R}^2 \to \mathbb{R}$ is a function, then the *lift* of γ to the graph of f is a new parameterized curve $\alpha : \mathbb{R} \to \mathbb{R}^3$ in 3-space defined by $\alpha(t) = (\gamma_1(t), \gamma_2(t), f(\gamma_1(t), \gamma_2(t)))$.

Reading: CM 17.1-3

1. Let P be a point in $\mathbb{R}^3$ and let $\vec{v}$ be a direction vector at P. Find a parameterization of the line through P in the direction $\vec{v}$ and with constant speed 1. (Hint: Look at the section "Unit vectors" in chapter 13.1, page 692.) How many other parameterizations of this line exist with constant speed 1?

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = 9 - 2x - 3y$. Let $P = (1, 2, 1)$.

a) For each angle θ from 0 to 2π, find a parameterization $\gamma_\theta : \mathbb{R} \to \mathbb{R}^2$ for the line starting at $(1, 2)$ in the x-y-plane at time $t = 0$, and heading out at an angle θ from x-axis with constant speed 1.



b) For each θ, let α_θ be the lift of your γ_θ to the graph of f. Write $\alpha_\theta(t)$.

c) Consider the parameterized curve $\delta : \mathbb{R} \to \mathbb{R}^3$ (considered with variable θ) defined by the "unit length lifts" $\delta(\theta) = \alpha_\theta(1)$. Explain why δ is an ellipse (hint: realize it as the lift to Γ_f of the unit circle around $(1, 2)$ in the in the x-y-plane; also ask yourself "what kind of surface is Γ_f?").

d) At what compass angle (with respect to the x-axis) do you have to start moving in to achieve the greatest instantaneous ascent on the graph at the point $(1, 2, 1)$ (hint: use the gradient).

e) For each θ, calculate the velocity vector $\vec{\alpha}'_\theta(0)$ (you should get a vector depending on θ).

f) For each θ, calculate the speed $\|\vec{\alpha}'_\theta(0)\|$ (this should be a function of θ).

g) **(Extra credit)** Use your single variable calculus prowess (or a computer!) to find the angle $0 \leq \theta \leq 2\pi$ giving the maximum value for this speed. Give an exact value and a decimal approximation for this angle.

h) **(Extra credit)** Compare the angles you got in parts *d)* and *g)* (they should differ by about π). Do they differ by exactly π? Explain what's going on here, perhaps draw a picture to help you explain.

3. CM 17.1 Problems 48, 68.

4. CM 17.2 Problems 28, 29, 35, 37.

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