Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Spring 2010

Problem Set # 4 (due Wednesday 24 February 2010)

Recall: If $\gamma : \mathbb{R} \to \mathbb{R}^2$ is a parameterized curve in the x-y-plane given by $\gamma(t)$ $(\gamma_1(t), \gamma_2(t))$, and $f : \mathbb{R}^2 \to \mathbb{R}$ is a function, then the *lift* of γ to the graph of f is a new parameterized curve $\alpha : \mathbb{R} \to \mathbb{R}^3$ in 3-space defined by $\alpha(t) = (\gamma_1(t), \gamma_2(t), f(\gamma_1(t), \gamma_2(t)))$.

In CM 17.2, there's a formula for the length of a segment of a parameterized curve. If $\beta : \mathbb{R} \to \mathbb{R}^n$ is any parameterized curve in n-space, and if $a \leq b$ are real numbers, then we have:

$$(\text{length of } \beta \text{ from } t = a \text{ to } t = b) = \int_a^b \|\beta(t)\|\, dt$$

Since for each t, $\|\beta(t)\|$ is a number, the integral above is just a standard single-variable definite integral.

Reading: CM 17.1-2

1. Let $P \in \mathbb{R}^3$ and let $\vec{v}$ be a direction vector at P. Find a parameterization of the line through P in the direction $\vec{v}$ and with constant speed 1.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = xy$. Let $P = (1, 2, 2)$.

(1) For each angle θ from 0 to 2π, find a parameterization γ_θ for the line starting at $(1, 2)$ in the x-y-plane at time $t = 0$, and heading out at an angle θ from the horizontal with constant speed 1.
(2) For each θ, let α_θ be the lift of your γ_θ to the graph of f. Write $\alpha_\theta(t)$.
(3) As a function of θ, calculate the length of α_θ from $t = 0$ to $t = 1$. This is the length travelled on the graph of f in one time unit walking at a compass angle θ.
(4) For which θ is the length of this path maximal/minimal?
(5) Find the compass angle you have to start walking in from P to achieve the greatest ascent/descent. Do these compare with the previous part?

3. CM 17.1 Exercises 14, 24, 26.
Problems 44, 48, 68.

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