Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Spring 2010

(Final) Problem Set # 10 (due Friday 23 April 2010)

Reading: CM 16.4-7

1. CM 16.7 Exercise 2, 4, 6
Problems 16, 20, 26,

2. CM 16 Review Problems 26, 28, 30, 32, 48,

3. Find the volume of the region in $\mathbb{R}^3$ between the surfaces $z = x^2 + y^2$ and $z = (x^2 + y^2 + 1)/2$.

4. EC* Sketch and find the volume of the region between the surfaces $x^2 + y^2 = 4z$ and $x^2 + y^2 + z^2 = 5$.

5. EC* Define

$$R_1 = \{(\rho, \theta, \phi) \,:\, 0 \leq \theta \leq 2\pi,\, 0 \leq \phi \leq \pi/2,\, 0 \leq \rho \leq \cos(\phi)\}$$

and

$$R_1 = \{(\rho, \theta, \phi) \,:\, 0 \leq \theta \leq 2\pi,\, 0 \leq \phi \leq \pi/2,\, 0 \leq \rho \leq \sin(\phi)\}$$

and let Φ be the spherical coordinate mapping. Sketch $\Phi(R_1)$ and $\Phi(R_2)$ and compute their volumes.

6. EC* Let $T\mathbb{R}^3$ be the solid torus (i.e. doughnut) formed by spinning around the z-axis a disk of radius r centered at $(a, 0, 0)$ in the x-z-plane. Of course, assume $0 < r < a$ otherwise you don't get a torus. Compute the volume of T. Hint: try cylindrical coordinates.

7. EC* Prove the *inflation principle*: if a region $R \subset \mathbb{R}^3$ has volume V, then for $a > 0$ the region aR of all points of R scalar multiplied by a, has volume a^3V. Hint: use the change of coordinates theorem.