Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Fall 2011

Problem Set # 9 (due Friday 04 November 2011)

Reading: CM 16.4-5.

1. CM 16.4 Exercise 6, 8, 12, 14, 16
Problems 20, 22, 34

2. CM 16.5 Exercises 12, 13, 14, 18 (place these regions anywhere you find most convenient in $\mathbb{R}^3$)
Problems 32 (only parts a and b), 33, 38, 40

3. (Extra credit) During the course of this problem, you will compute the "improper integral"

$$G = \int_{-\infty}^{\infty} e^{-x^2}\, dx$$

which is defined as the following limit

$$\lim_{r\to\infty} \int_{-r}^{r} e^{-x^2}\, dx.$$

Let $G(r) = \int_{-r}^{r} e^{-x^2}\, dx$ for any $r \geq 0$.

a) Let $f(x,y) = e^{-x^2-y^2}$ and $R(r)$ be the box with corners $(r,-r)$, (r,r), $(-r,r)$, and $(-r,-r)$. Use Fubini's Theorem to show that $G(r)^2 = \int_{R(r)} f$.

b) Let $C(r)$ be the disk of radius r centered at the origin. Show that

$$\int_{C(r)} f \leq \int_{R(r)} f \leq \int_{C(\sqrt{2}r)} f.$$

c) Change to polar coordinates to compute $\int_{C(r)} f$ and $\int_{C(\sqrt{2}r)} f$.

d) Now calculate the limits

$$\lim_{r\to\infty} \int_{C(r)} f, \qquad \text{and} \qquad \lim_{r\to\infty} \int_{C(\sqrt{2}r)} f.$$

e) From this, what do you conclude is the value of $\lim_{r\to\infty} G(r)^2$.

f) Finally, what do you conclude is the value of G?

Note: to make the final two steps really rigorous, you'll have to take Math 411 (or they might cover it in Math 250, depending on the instructor)!

Emory University, Department of Mathematics & CS, 400 Dowman Dr NE W401, Atlanta, GA 30322
E-mail address: auel@mathcs.emory.edu