Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Fall 2011

Problem Set # 1 (due Friday 02 Sep 2011)

Notations: If S is a set of elements (points, numbers, elephants, ...) then the notation "$s \in S$" means "s is an element of the set S." If T is another set, then the notation "$T \subset S$" means "every element of T is an element of S" or "T is a *subset* of S." For example, the set of squares is a subset of the set of rectangles.

Recall that we have notations for the following sets:

- $\mathbb{R}$ is the real 1-dimensional "number line"
- $\mathbb{R}^2$ is the real 2-dimensional "Cartesian plane"
- $\mathbb{R}^3$ is the real 3-space
- $\mathbb{R}^n$ is the real n-space.

We can express a point $v \in \mathbb{R}^n$ as an n-tuple $(x_1, \dots, x_n)$ of real numbers.

A *multivariable mapping* $f : \mathbb{R}^n \to \mathbb{R}^m$ is a recipe, which given any element $v \in \mathbb{R}^n$, produces an element $f(v) \in \mathbb{R}^m$. For certain cases of n and m such mappings have other names:

- $n = 1$ and $m = 1$, called *single variable function*,
- $n = 1$ and $m > 1$, called *parametrized curve (in $\mathbb{R}^m$)*,
- $n > 1$ and $m > 1$, called *vector field*,
- $n \geq 1$ and $m = 1$, called *function (on $\mathbb{R}^n$)*.

More generally, given subsets $V \subset \mathbb{R}^n$ and $W \subset \mathbb{R}^m$, a (multivariable) mapping $f : V \to W$ is a recipe, which given any element $v \in V$ produces an element $f(v) \in W$. We call V the *domain* and W the *codomain* of f. The *image* $\mathrm{im}(f) \subset W$ of f is the subset of the codomain consisting of all elements "hit by f" i.e. all elements $w \in W$ such that $w = f(v)$ for some $v \in V$ in the domain.

Definition. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a multivariable mapping, then the *graph* of f is the subset Γ_f : $\mathbb{R}^{n+m}$ consisting of all points $(v, f(v)) \in \mathbb{R}^{n+m}$ where $v \in \mathbb{R}^n$, i.e. all points $(x_1, \dots, x_n, f_1(x), \dots, f_m(x))$ where we write $v = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $f(v) = (f_1(v), \dots, f_m(v)) \in \mathbb{R}^m$.

1. *Graphs of Multivariable Functions*

a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function on $\mathbb{R}^2$. Give $\mathbb{R}^3$ the standard (x, y, z) coordinates. Describe the intersection of Γ_f with the x-y-plane by an implicit equation in terms of the function f. Draw this set for $f(x, y) = x^2 - xy$.

b) Now let $f(x, y) = x^2 + y$. Draw the intersection of Γ_f with the x-z-plane and find both an implicit equation and a parameterization describing it.

c) Again, let $f(x, y) = x^2 + y$. For each $\theta \in [0, \pi]$, let P_θ be the plane through the z-axis and the point $(\cos\theta, \sin\theta, 0)$. Describe (in words and by writing both an implicit equation and a parameterization) the intersection of P_θ and Γ_f for each θ (descriptions will depend on θ).

2. CM Problem 12.1.29 (it turns out to be a surface in $\mathbb{R}^3$, write an implicit equation for it)

3. CM Problems 12.2.15 and 12.2.24

4. CM Exercise 12.3.14 and Problem 12.3.33

5. CM Problem 12.5.30

6. CM Problem 13.1.33

7. CM Problems 13.3.37, 13.3.39, and 13.3.40

8. CM Problems 13.4.25 and 13.4.28

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