UNIVERSITY OF PENNSYLVANIA DEPARTMENT OF MATHEMATICS
Math 170 Ideas in Mathematics Summer Session I 2007
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Homework #4 (due Tuesday 19 June 2007)

1. Corruption and golf. Express the following propositions using the standard logical operators $\wedge, \vee, \sim$, and $\Rightarrow$ and the quantifiers $\exists$ and $\forall$. Let the universe of the quantifiers be "all US government officials" and define the following proposition forms

$$\begin{aligned} S(x) &= \text{“}x \text{ is a senator”} \\ G(x) &= \text{“}x \text{ has played golf”} \\ C(x) &= \text{“}x \text{ is corrupt”} \end{aligned}$$

to help you out.

a) Some senators have played golf and are corrupt.

b) Some senators have played golf only if they are corrupt.

c) Any senator, that is corrupt, has played golf.

d) Any senator, that has played golf, is corrupt.

e) No senator has played golf unless she's corrupt.

f) Any senator has played golf if and only if he's corrupt.

g) Senators who've golfed are all corrupt.

h) Only corrupt senators have ever golfed.

i) Only senators who've golfed are corrupt.

j) It's only senators, who've golfed if they're corrupt.

k) Some senators have golfed even if they're not corrupt.

l) If a government official is corrupt, then he's a senator who has played golf.

m) Some senators are corrupt and haven't golfed.

n) Some senators are neither corrupt nor have ever golfed.

o) No senator who is corrupt has never played golf.

p) Any senator has either played golf or is corrupt.

q) Some senators are corrupt whether or not they've played golf.

r) No senator provided he's never golfed is not corrupt.

For more information, read the January 22nd, 2006, New York Times style desk article "The Muddying of the Greens" by Anne E. Kornblunt.

2. Some rules of logic. Use the method of truth tables to prove the following rules of logic, letting P, Q and R be proposition statements:

a) Absorption laws:

$$\begin{aligned} P \wedge (P \vee Q) &\equiv P \\ P \vee (P \wedge Q) &\equiv P \end{aligned}$$

b) De Morgan's laws:

$$\begin{aligned} \sim (P \wedge Q) &\equiv \sim P \ \vee \sim Q \\ \sim (P \vee Q) &\equiv \sim P \ \wedge \sim Q \end{aligned}$$

c) Distributivity laws:

$$\begin{aligned} P \wedge (Q \vee R) &\equiv (P \wedge Q) \vee (P \wedge R) \\ P \vee (Q \wedge R) &\equiv (P \vee Q) \wedge (P \vee R) \end{aligned}$$

3. Fallacies Write each of the following exchanges using the logical operators and decide if the final "deductions" are valid.

a) If my grandfather is smoking a pipe then he's reading the newspaper.
Right now my grandfather is smoking a pipe,
so he must be reading the newspaper.

b) *Professor:* If you do reasonably well in this class then you'll get an A.
Parent to student: So how did you do in that class?
Student to parent: Well, I got an A in the class so I must have done reasonably well.

c) *Your date:* Hey, can I come over?
You: Well, my house is always clean on Sunday.
Your date: But it's Tuesday night.
You: Ohh, so no then, my house will be dirty.

d) *Your date:* I want to either eat ice cream at the movie theater or eat ice cream in front of a DVD.
You: So you want to eat ice cream and either go to the movie theater or rent a DVD?

e) Today, I need to go play basketball and fix my bicycle, or I just need to fix my bicycle.
I guess I'll go play basketball.

f) *Patron to waiter:* I'll take the chef's special as long as it has wild mushrooms or comes with arugula but if it's at all spicy then it better come with arugula.
Waiter to chef: Hey, make something with arugula or make something with wild mushrooms that's not spicy!

4. Smoking. Convert the following common parental exchange into propositions using the logical operators, quantifiers, and convenient propositional forms. Then analyze the mom's argument for why her kid shouldn't smoke. Does it hold up? Can the kid say anything to counter her?

Mom: You should stop smoking.
Kid: But all my friends are smoking.
Mom: If all your friends jumped off a cliff, should you jump off the cliff?
Kid: Of course not, mom.
Mom: Then you shouldn't smoke, now should you?