UNIVERSITY OF PENNSYLVANIA DEPARTMENT OF MATHEMATICS
Math 170 Ideas in Mathematics Summer Session I 2007
Instructor Asher Auel

Homework #3 (due Tuesday 12 June 2007)

1. FAPP In For All Practical Purposes (FAPP):

a) Ed. 6, Chapter 19, exercises 38, your choice of 37 or 40, and from Ed. 7 handout: 40, your choice of 38 or 39

or

Ed. 7, Chapter 19, exercises 34, your choice of 33 or 36 (refer to Figure 19.12), 40, your choice of 38 or 39

b) Ed. 6, Chapter 20, exercises 7, 8, 15, 16

or

Ed. 7, Chapter 20, exercises 12, 13, 29, 30.

2. Symmetry groups.

a) Draw a plane figure with symmetry group isomorphic to the additive group of integers $(\mathbb{Z}, +)$. Explain.

b) Find the symmetry group of a square and write out the multiplication table.

3. A group under multiplication. Recall that the set of numbers $\{0, 1, 2, \ldots, 11\}$ forms a group under the operate $+_{12}$ of addition then taking the remainder mod 12:

$$a +_{12} b = a + b \mod 12,$$

and this group is called $(\mathbb{Z}/12/Z, +_{12})$, otherwise known as the *clock group*.

a) We'll define another binary operation, $\cdot_{12}$, on the set $\{0, 1, \ldots, 11\}$ by multiplying and then taking the remainder mod 12:

$$a \cdot_{12} b = ab \mod 12.$$

Show that $\{0, 1, \ldots, 11\}$ with this operation does not form a group. What happens?

b) Thinking of 1 as an "identity" under the operation $\cdot_{12}$, find all numbers from 0 to 11 which are invertible under $\cdot_{12}$. Hint: There are four of them.

c) Show that the set of "invertible" numbers from part *b)* forms a group under the operation $\cdot_{12}$. This group is called the *multiplicative group* $((\mathbb{Z}/12\mathbb{Z})^\times, \cdot_{12})$. Write its multiplication table.

d) Can the group $((\mathbb{Z}/12\mathbb{Z})^\times, \cdot_{12})$ be generated by a single element?

e) Draw a shape that has symmetry group isomorphic to $((\mathbb{Z}/12\mathbb{Z})^\times, \cdot_{12})$.