

# MATH 170 IDEAS IN MATHEMATICS (SUMMER 2006)

## Problem Set 7: Peeking into *TNT*.

Due in class Thursday, June 8th

### 1. A bit more *PC*

Answer the following questions:

a. For the following strings, decide if they are theorems of *PC*:

- $\langle \langle \langle P \Rightarrow Q \rangle \Rightarrow Q \rangle \Rightarrow \langle \neg P \Rightarrow Q \rangle \rangle$
- $\langle \langle \langle P \Rightarrow Q \rangle \wedge \langle P \Rightarrow \neg Q \rangle \rangle \Rightarrow \neg P \rangle \rangle$

The last string is a statement usually called *reductio ad absurdum*. Why?

b. Prove the following (axiom-like) metatheorems about *PC*. If  $x$  and  $y$  are any well-formed strings, then

- if  $x$  is any well-formed string then  $\langle x \vee \neg x \rangle$  is a theorem
- if  $x$  is a theorem and  $y$  is any well-formed string then  $\langle x \vee y \rangle$  is a theorem
- if both  $\neg y$  and  $\langle x \Rightarrow y \rangle$  are theorems then so is  $\neg x$
- if both  $\neg x$  and  $\langle x \vee y \rangle$  are theorems then so is  $y$
- $\langle x \wedge y \rangle$  is interchangeable with  $\langle y \wedge x \rangle$
- $\langle x \vee y \rangle$  is interchangeable with  $\langle y \vee x \rangle$
- $\neg \langle x \wedge y \rangle$  is interchangeable with  $\langle \neg x \vee \neg y \rangle$

### 2. Beginning *TNT*

Note: the set of all non-negative integers  $0, 1, 2, \dots$  will be denoted the *natural numbers*. Note that the natural numbers are the “universe” for the quantifiers  $\forall$  and  $\exists$  in *TNT*.

Answer the following:

a. Express the following sentences in the language of *TNT* as bounded strings:

- Every natural number is equal to 3.
- No natural number squared is equal to itself.
- No natural number plus 1 is equal to itself.
- There exist even natural numbers.
- There exist odd natural numbers.
- No natural number is both even and odd.
- If a natural number is odd then it plus 1 is even.

Which of these bounded strings are true?

b. Express the following sentences in the language of *TNT* as unbounded strings:

- $a$  plus 3 is an odd number.
- $a$  is a prime number.
- $a$  is divisible by 5.
- $a$  is not divisible by 7.
- $a$  has remainder 1 when divided by 3.
- $a$  is a power of 2.