MATH 170 IDEAS IN MATHEMATICS (SUMMER 2006)

Problem Set 4: More symbolic logic.

Due in class Tuesday, May 30th

1. More rules of logic

As always, let P and Q be propositions (i.e. statements that are either true or false). Recall the symbolic logic notation $P \Rightarrow Q$ for "if P then Q" or "P implies Q," and also recall its corresponding truth table:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

where as usual, T stands for "true" and F for "false." Also let $P \equiv Q$ stand for the metalogical statement P is logically equivalent to Q, i.e. P and Q have the same truth tables.

Use the method of truth tables (from Problem Set 3) to prove the following rules of logic:

a. Rule of contrapositive or *modus tollens*:

$$P \Rightarrow Q \quad \equiv \quad \neg Q \Rightarrow \neg P$$

b. Absorption laws:

$$P \wedge (P \vee Q) \quad \equiv \quad P$$
$$P \vee (P \wedge Q) \quad \equiv \quad P$$

c. Distributivity laws:

$$P \wedge (Q \vee R) \quad \equiv \quad (P \wedge Q) \vee (P \wedge R)$$
$$P \vee (Q \wedge R) \quad \equiv \quad (P \vee Q) \wedge (P \vee R)$$

where R is an additional proposition.

(Hint: in **b.** and **c.** if you prove the first statement using truth tables, you can use the properties of $\neg$ from Problem Set 3 to prove the second more easily.)

2. Fallacies

Write each of the following logical "arguments" in logical symbols and decide if the "deductions" are valid.

a. If my grandfather is smoking a pipe then he's reading the newspaper.
Right now my grandfather is smoking a pipe,
so he must be reading the newspaper.

b. If you do reasonably well in this class then you'll get an A.
I got an A in this class,
therefor I did reasonably well in this class.

c. My kitchen is always clean on Sunday.
Today is Tuesday,
so my kitchen is dirty.

d. I want to either eat ice cream at the movie theater or eat ice cream in front of the DVD.
I want to eat ice cream and either go to the movie theater or rent a DVD.

e. I need to fix my bicycle and either go play basketball or fix my bicycle.
I guess I'll go play basketball.

3. A FIGURE-FIGURE sequence of numbers?

On page 73 of *GEB*, Hofstadter asks you if you can characterize the set of integers (or its negative space):

$$1,\ 3,\ 7,\ 12,\ 18,\ 26,\ 35,\ 45,\ 56,\ 69,\ \ldots$$

He also asks how this sequence is like the FIGURE-FIGURE picture on page 69. Answer his questions.

4. A modified pq-system and $x + y \geq z$

Can you think of a way to modify Hofstadter's original pq-system (either by adding new axioms or new rules of production), so that the interpretation

$$\begin{aligned} \mathsf{p} &\longleftrightarrow + \\ \mathsf{q} &\longleftrightarrow \geq \\ - &\longleftrightarrow 1 \\ -- &\longleftrightarrow 2 \\ --- &\longleftrightarrow 3 \\ &\vdots \end{aligned}$$

will make an isomorphism (i.e. will be consistent and complete) with the set of truths of the form $x + y \geq z$ for positive integers x, y, and, z?