Dartmouth College Department of Mathematics
Math 101 Topics in Algebra: Quadratic Forms Syllabus
Fall Term 2020

Professor:	Asher Auel	**Lecture:**	Remote
Office:	Remote	**Time:**	MWF 10:10 – 11:15 am Th 12:15 – 1:15 pm X-hour
Text:	See Website.		
Web-site:	http://math.dartmouth.edu/~auel/courses/101f20/		

Introduction: This course will provide an introduction to the algebraic theory of quadratic forms over fields. Topics will include the elementary invariants (discriminant and Hasse invariant), classification over various fields (real numbers, finite fields, p-adic numbers, rational numbers, and rational function fields), local-global principals (e.g., Hasse–Minkowski and the Milnor exact sequence), isotropy and Witt decomposition, Witt groups, Pfister forms, and Milnor K-theory. Along the way, we will also cover the basic theory of complete discretely valued fields, orthogonal groups, and lattices. Special topics might include sums of squares (e.g., Lagrange's theorem and Pfister's bound for the Pythagoras number), the Milnor conjectures, the u-invariant, and trace forms, depending on the interests of the participants.

Grading: Your final grade will be based on homework, a midterm exam, and a final exam.

Prerequisites: Prior experience with linear algebra will be necessary and prior experience with field and Galois theory will be helpful but not necessary.

Topics covered: Subject to change.

(1) Background material. Bilinear forms. Quadratic forms and symmetric bilinear forms. Diagonalization. Square classes and quadratic extensions. Binary and ternary forms.

(2) Elementary invariants. Discriminant and signed discriminant. Signature over ordered fields. Hasse-Witt invariant and Clifford invariant.

(3) Isotropy. Hyperbolic spaces. Witt group and Witt equivalence. Cancellation.

(4) Fundamental filtration of the Witt group. Pfister forms. Multiplicative forms.

(5) Discrete valuations. Complete discretely valued fields. Inverse limits. Completions. Ostrowski's theorem.

(6) Symbols. Residue maps and the tame symbol. Springer's theorem. Hilbert symbol. Hilbert reciprocity.

(7) Classification over the real numbers (Sylvester's law of inertia). Classification over finite fields. Classification over p-adic numbers. Hasse-Minkowski and classification over the rational numbers.

(8) Lattices. Discriminants and sublattices. Even lattices. Discriminant form. Overlattices. Saturation.

(9) (Optional.) Sums of squares. Lagrange's theorem. Pythagoras numbers. Pfister's bounds and recent results.

(10) (Optional.) Orthogonal groups. Reflections. Eichler transvections.

(11) (Optional.) Milnor conjectures.

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