Math 8 Final Exam Practice Problems

Disclaimer: These problems are meant to help you practice for the exam on the material that has been covered since the second midterm exam. You already have the two midterm exams and two previous practice exams to help study the other material in the course. These are in no way a comprehensive set of problems - you may well see completely different types of problems on the final exam.

1. Find a vector tangent to the curve of intersection of the two surfaces $z = x^2 - y^2$ and $xyz + 30 = 0$ at the point $(-3, 2, 5)$.

2. Let $f(x, y, z) = ye^{-x^2}\sin(z)$. Find the equation of the tangent plane to the level surface of f at the point $(0, 1, \pi/3)$.

3. Let $w = e^{xy}, x = f(s, t), y = g(s, t)$. Suppose also that

$$f(0,2) = 1, f_s(0,2) = 3, f_t(0,2) = 5, g(0,2) = 2, g_s(0,2) = -1, g_t(0,2) = 0.$$

   a.) Find $\frac{\partial w}{\partial s}$ at the point $(s, t) = (0, 2)$.
   b.) Calculate the directional derivative of f in the direction given by $\vec{v} = \langle 3, 4\rangle$.

4. Consider the sphere $x^2 + y^2 + z^2 = r^2$ and the elliptic cone $z^2 = a^2x^2 + b^2y^2$, where a, b, r are constants. Show that at every point of their intersection, the corresponding tangent planes are orthogonal.

5. A ball is placed at the point $(1, 2, 3)$ on the surface $z = y^2 - x^2$. Give the direction in the xy-plane that the ball will start to roll.

6. Let $f(x, y) = x^4 + y^4 + x^2 - y^2$.
   a.) Find and classify all the critical points of f.
   b.) Use the method of Lagrange multipliers to find the largest and smallest values of f on the circle $x^2 + y^2 = 4$.

7. Find the extreme values of the function $f(x, y) = x^2ye^{-(x+y)}$ on the triangular region given by $x \geq 0, y \geq 0$ and $x + y \leq 4$.

8. The temperature at the point (x, y, z) is given by $T(x, y, z) = xy^2z$. Find the direction of maximum increase in temperature at the point $(1, -2, 3)$. If you move on a straight line so that your velocity as you pass through the point $(1, -2, 3)$ is $\vec{v} = \langle 2, 1, 2\rangle$ then what is the rate of temperature increase as you pass through $(1, -2, 3)$?

9. Find the absolute extreme values of the function $f(x, y) = \frac{3}{2}x^2 + x + y^2 - 2$ on the closed disk $x^2 + y^2 \leq 4$.