1. (5 points) Does the integral
$$\int_0^\infty xe^{-x^2}\,dx$$
converge? If so, to what value does it converge?

2. (5 points each) Which of the following series converge or diverge? Justify your answers:

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{e^n}$$

(b)
$$\sum_{n=0}^{\infty} \frac{3^n}{(2n+1)!}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{n(n+1)^2}$$

$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{n(n+1)^2}$$

3. (10 points) Find a power series representation for

$$f(x) = \arctan(3x)$$

around $x = 0$ and determine its radius of convergence.

4. (25 points total) Let $f(x, y) = x \sin(y)$

    (a) (5 points) Find $f_x, f_y, f_{xx}, f_{yy}, f_{xy}$.

    (b) (5 points) What are the critical points of f?

(c) (5 points) Classify the critical points of f.

(d) (10 points) Find the absolute maximum and minimum of f on the region given by $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

5. (15 points total) The temperature at a point (x, y) is given by

$$T(x, y) = 200e^{-x^2 - 3y^2}$$

where T is measured in degrees centigrade and x, y in meters.

(a) (5 points) Find the rate of change of the temperature at the point $(2, -1)$ in the direction toward the point $(3, -3)$.

(b) (5 points) At this same point, in which direction does the temperature change the fastest?

(c) (5 points) What is the maximal rate of increase at this point?

6. (15 points total) Let S be the surface given by $z = f(x, y)$ where $f(x, y) = \sqrt{4 - x^2 - 2y^2}$

   (a) (5 points) Find ∇f.

   (b) (5 points) What is the equation of tangent plane to S at the point $(1, -1, 1)$?

(c) (5 points) Use a linear approximation of this function at $(1, -1, 1)$ to approximate the value of the function f at the point $(1.1, -1.2)$.

7. (15 total)

(a) (10) Evaluate
$$\int \sec^3(\theta)\, d\theta$$

Hint: use integration by parts

(b) (5) Evaluate the integral
$$\int_0^1 \sqrt{1 + s^2} \, ds$$

(c) (5) Find the arclength of $\vec{r}(t) = \left\langle t, \frac{t^2}{2} \right\rangle$ for $0 \leq t \leq 1$.

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NAME (Print!): _____

Math 8

3/13/2005
Final Exam

Problem	Points	Score
1	5	
2	15	
3	10	
4	25	
5	15	
6	15	
7	15	
Total	100	