Math 3 Fall 2014

Final Exam

November 21, 2014

Your name (please print): ____

Circle your section: Pauls Cai Andrews Hein

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. Except for the multiple choice questions, you must justify all of your answers to receive credit - please write clearly in complete sentences. You may not give or receive any help on this exam and all questions should be directed to your instructor.

You have **3 hours** to work on all **24** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5-24	40	
Total	**100**	

Answer Sheet for Multiple Choice Questions

Problem	Answer
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	

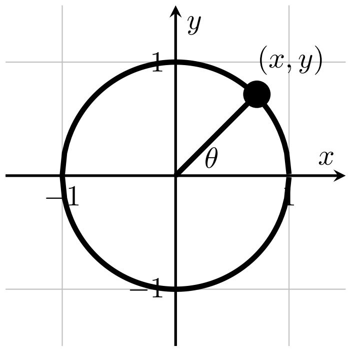
Note that each multiple choice question is worth 2 points. Thus, it is a good strategy to first try to complete the long answer questions (which together are worth 60 points) before answering the multiple choice questions.

1. Let $f(x) = \displaystyle\int_1^x \frac{1}{(t-2)^3}\,dt.$

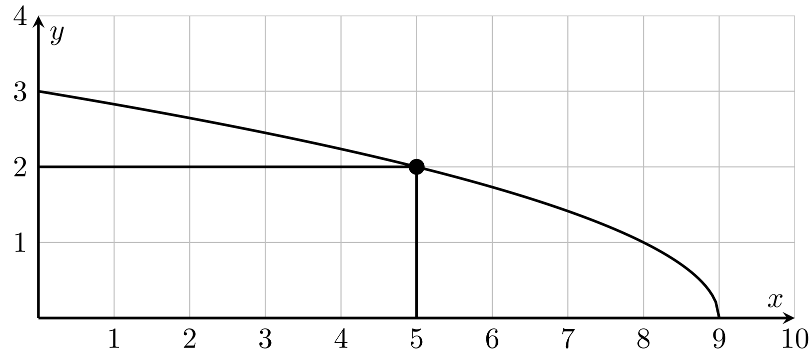
   (a) Use the Fundamental Theorem of Calculus to solve the integral defining $f(x)$.

   (b) Using the solution you found in part (a), compute $\displaystyle\lim_{x\to\infty} f(x)$.

2. An object is traveling around the unit circle at a constant rate with $\frac{d\theta}{dt} = \frac{\pi}{2}$. What is $\frac{dx}{dt}$ when $\theta = \frac{\pi}{4}$?

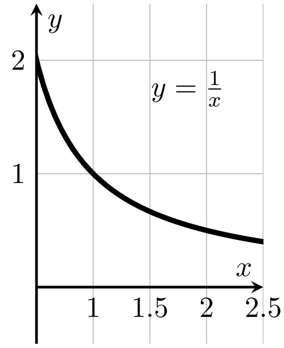


3. Let R be a rectangle with one side on the positive x-axis, one side on the positive y-axis, and its top right corner on the curve $x = 9 - y^2$. What is the maximum perimeter that R can have?



4. Determine the following:

(a) Estimate the area under the curve of the function $f(x) = \frac{1}{x}$ on the interval $[1, 2]$ using the left endpoint method with 4 rectangles **(you do not need to simplify fractions).**



(b) Is this an underestimate, overestimate, or is there ambiguity?

5. Let $f(x) = \int_x^{-2} \frac{\cos t}{t} dt$, where $x < 0$. What is $f'(x)$?

(a) $f'(x) = \frac{\cos x}{x}$.

(b) $f'(x) = -\frac{\cos x}{x}$.

(c) $f'(x) = \frac{-x \sin x - \cos x}{x^2}$.

(d) $f'(x) = \frac{x \sin x + \cos x}{x^2}$.

(e) $f(x)$ is not differentiable on the interval $(-\infty, 0)$.

6. The function $f(x) = \frac{x^4}{4} + \frac{x^3}{3} - x^2 + 5$ has

(a) local maxima at $x = -2$ and $x = 1$ and a local minimum at $x = 0$.

(b) local minima at $x = -2$ and $x = 1$ and a local maximum at $x = 0$.

(c) local maxima at $x = -2$ and $x = 1$ and an inflection point at $x = 0$.

(d) local minima at $x = -2$ and $x = 1$ and an inflection point at $x = 0$.

7. Which of the following statements is true?

(a) $\frac{d}{dx}\big(\tan(x^2 + x + 1)\big) = (2x + 1)\sec^2(x^2 + x + 1)$

(b) $\frac{d}{dx}\left(e^{x^2}\right) = e^{x^2}$

(c) $\frac{d}{dx}(x \ln(x)) = 1 + x$

(d) $\frac{d}{dx}\left(\frac{x}{x^3 + 1}\right) = \frac{1 - 3x^3}{(x^3 + 1)^2}$

8. What is $\int_1^{e^3} \frac{(\ln x)^2}{x} dx$?

(a) 0

(b) 3

(c) 6

(d) 9

(e) 12

9. What are the critical point(s) of the function

$$f(x) = \int_0^x \frac{t^2 + 3t - 4}{t^2 + 2} dt?$$

(a) $x = 0$.

(b) $x = -\frac{3}{2}$.

(c) $x = -4$ and $x = 1$.

(d) $x = 4 + \sqrt{6}$ and $x = 4 - \sqrt{6}$.

(e) $f(x)$ does not have any critical points.

10. Find the equation of the tangent line to the graph of the function $f(x) = \cos(x) + 2e^x$ at the point $(0, 3)$.
    (a) $y = x + 3$.
    (b) $y = 2x + 3$.
    (c) $y = x$.
    (d) $y = 2x$.
    (e) The function $f(x)$ is not differentiable at $x = 0$.

11. The function $f(x) = |x|x$ is
    (a) continuous at $x = 0$ because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) \neq f(0)$.
    (b) not continuous at $x = 0$ because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$.
    (c) continuous at $x = 0$ because $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$.
    (d) not continuous at $x = 0$ because $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$.
    (e) continuous at $x = 0$ because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$.

12. What are the vertical asymptote(s) of the function

$$f(x) = \frac{x^2 + x - 6}{x^2 + 7x + 12}?$$

    (a) $x = -4$.
    (b) $x = -3$.
    (c) $x = 2$.
    (d) $x = -4$ and $x = -3$.
    (e) $x = -3$ and $x = 2$.

13. Let $f(x) = e^x \tan(x)$; what is $f'(x)$?
    (a) $f'(x) = e^x + \sec^2(x)$
    (b) $f'(x) = e^x \sec^2(x) - e^x \tan(x)$
    (c) $f'(x) = e^x \tan(x)$
    (d) $f'(x) = e^x \sec^2(x)$
    (e) $f'(x) = e^x(\sec^2(x) + \tan(x))$

14. The current unit price for a new smartphone is $300.00 and the smartphone company can sell 2000 new phones per month. Market study shows that for each dollar decrease in the price, the monthly sales will increase by 20. Under such assumptions, what is the best unit price to maximize the company's total revenue?
    (a) $100.00
    (b) $150.00
    (c) $200.00
    (d) $250.00
    (e) $300.00

15. Find $\lim_{x\to\infty} (\sqrt{x^4 + 2x^3} - x^2)$:

(a) 0
(b) 1
(c) 2
(d) 4
(e) ∞.

16. Find the area of the region bounded by $y = 3x^2 - x^3$ and the x-axis.

(a) 6
(b) $\frac{13}{2}$
(c) $\frac{27}{4}$
(d) $\frac{49}{8}$
(e) $\frac{81}{16}$

17. If we use Riemann sums with rectangles whose heights are given using the left hand endpoints of their bases, then

$$\int_2^4 (x^2 + 1)\, dx =$$

(a) $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\left((i-1)\frac{2}{n} \right)^2 + 1 \right)$
(b) $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\left(2 + (i-1)\frac{2}{n} \right)^2 + 1 \right)$
(c) $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\left(2 + i\frac{2}{n} \right)^2 + 1 \right)$
(d) $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\left(i\frac{2}{n} \right)^2 + 1 \right)$

18. Let

$$g(x) = \int_{-55}^{3x} (10x^2 + 5x + 1)\ dx.$$

Then, the slope of the tangent line to the graph of $g(x)$ at $x = 0$ is

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

19.

$$\int x \sin(x^2 + 1)\ dx =$$

(a) $\frac{1}{2}\cos(x^2 + 1) + C$

(b) $-2\sin(x^2 + 1) + C$

(c) $x\sin(x^2 + 1) + C$

(d) $-\frac{1}{2}\cos(x^2 + 1) + C$

20. A particle travels along a straight path with velocity (in meters/sec) given by the function

$$v(t) = 3t^2 - 6t.$$

What is the total distance traveled between times $t = 0$ and $t = 4$?

(a) 16 meters.

(b) 24 meters.

(c) 48 meters.

(d) 64 meters.

21. Let $f(x)$ be a continuous function on the interval $[a, b]$ and suppose that $F(x)$ is an antiderivative of $f(x)$ on $[a, b]$. Choose the statement which is **false**.

(a) $\int f(x)\, dx = F(x) + C$ on the interval $[a, b]$.

(b) $F'(x) = f(x)$ for all $a < x < b$.

(c) $\frac{d}{dx}\int_a^x f(t)\, dt = F(x)$ for all $a \leq x \leq b$.

(d) $F(a) + \int_a^b f(x)\, dx = F(b)$.

22. The function

$$f(x) = \frac{2x-1}{\sqrt{x^2+3x}}$$

has

(a) no horizontal asymptotes.

(b) a single horizontal asymptote at $y = 2$.

(c) a single horizontal asymptote at $x = 0$.

(d) two horizontal asymptotes at $y = 2$ and $y = -2$.

23. Choose the correct statement.

(a) $\displaystyle\int \sin(x)\,dx = \cos(x) + C.$

(b) $\displaystyle\int \frac{1}{x^4}\,dx = -\frac{1}{5x^5} + C.$

(c) $\displaystyle\int 4e^{4x}\cos(e^{4x})\,dx = \sin(e^{4x}) + C.$

(d) $\displaystyle\int \frac{1}{4x+1}\,dx = \ln(4x+1) + C.$

24. If

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2, \end{cases}$$

then f is

(a) not differentiable for any values of m and b.

(b) differentiable when $m = 4$ and $b = -4$.

(c) differentiable when $m = -4$ and $b = 4$.

(d) differentiable when $m = 1$ and $b = 2$.

(e) differentiable when $m = 2$ and $b = 0$.

This page is for additional work.

This page is for additional work.

This page is for additional work.