For this page, let $f(x) = \dfrac{3x+7}{x-2}$.

1. What is $\lim_{x\to\infty} f(x)$?

   (a) 0
   (b) 3
   (c) 1
   (d) $\infty$
   (e) The limit does not exist.

2. What is the domain of $f(x)$?

   (a) $(-\infty, \infty)$
   (b) $(-\infty, -1) \cup (-1, \infty)$
   (c) $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$
   (d) $(-\infty, 2) \cup (2, \infty)$
   (e) $(-2, 1) \cup (1, \infty)$

For this page, let $f(x) = \dfrac{2x}{x^4 - 5x^2 + 6}$.

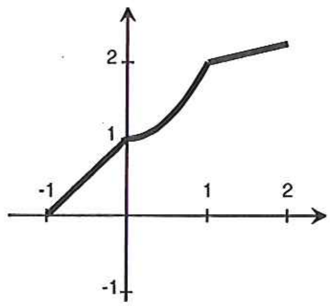
3. What are the horizontal asymptotes of $f$?

   (a) $y = 2/5$
   (b) $y = 3$ and $y = 2$
   (c) $y = 0$
   (d) $x = \pm\sqrt{3}$
   (e) none of the above

4. What are the vertical asymptotes $f$?

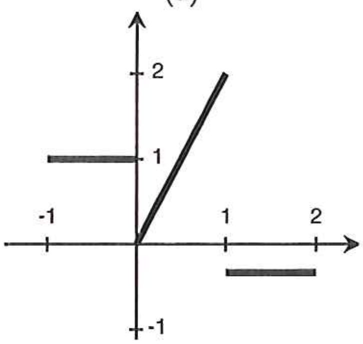
   (a) $x = 0$
   (b) $x = \sqrt{2}$
   (c) $x = \pm\sqrt{2}$ and $x = \pm\sqrt{3}$
   (d) $y = 2$ and $y = 3$
   (e) none of the above

Suppose the graph of the function $f(x)$ looks like

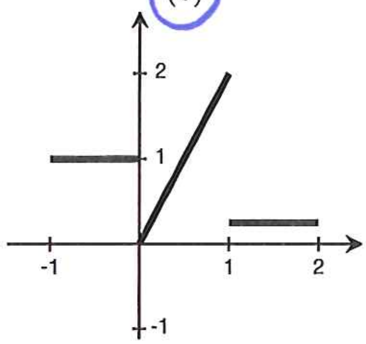



5. Of the following graphs, which could be the graph of $f'(x)$?

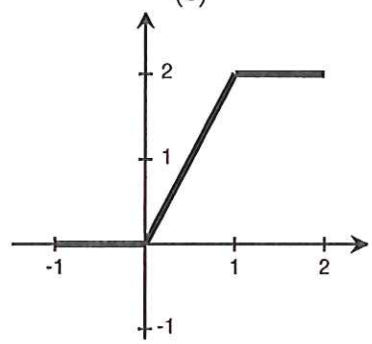
(a)



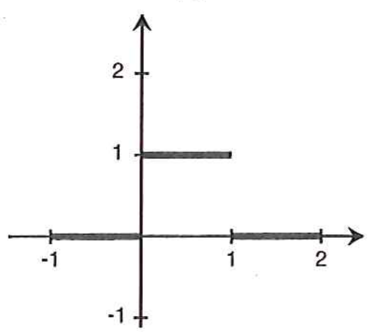

(b)



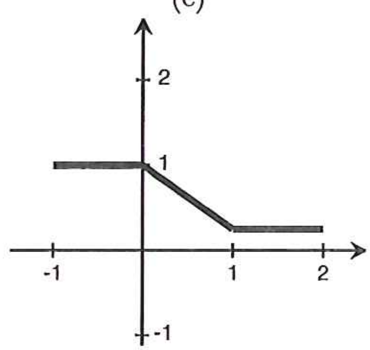

(c)



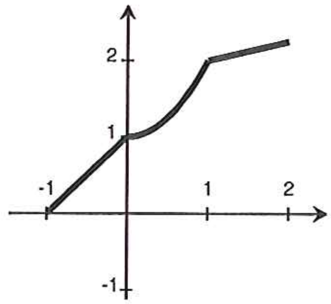

(d)




(e)

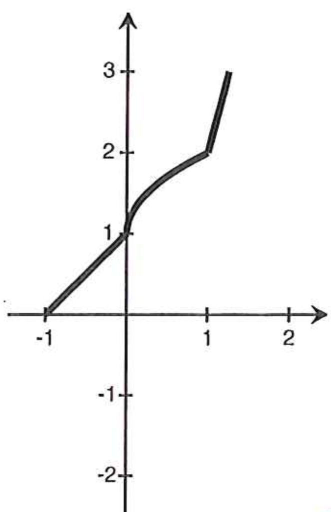


Again, suppose the graph of the function $f(x)$ looks like

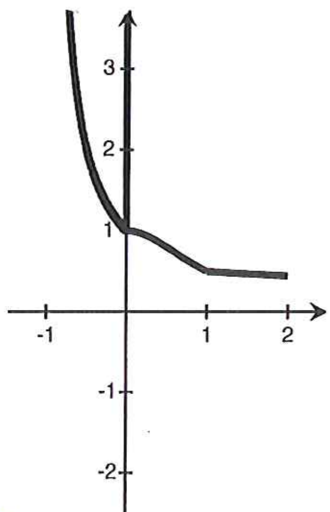



6. Of the following graphs, which is the graph of $f^{-1}(x)$?

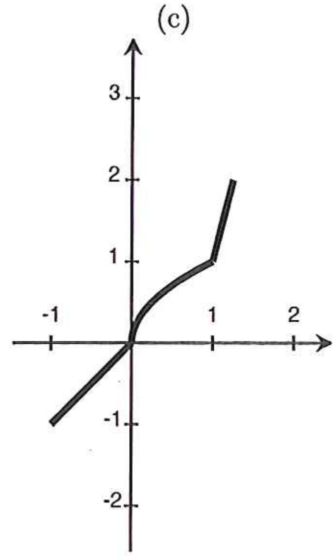
(a)



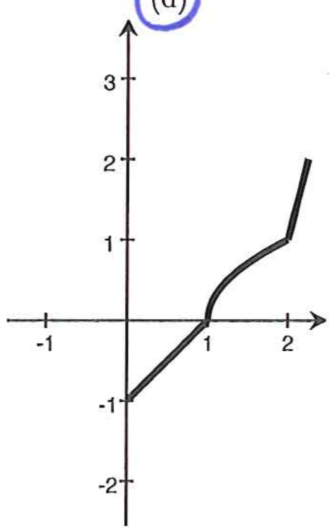

(b)



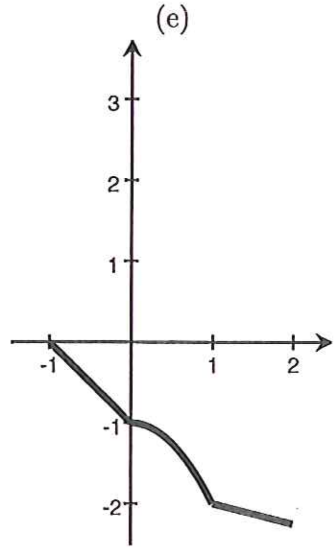

(c)




(d)




(e)



For this page, let $f(x) = \dfrac{1}{2x}$ and $g(x) = \dfrac{x-1}{x+1}$.

7. What is $f^{-1}(x)$?

   (a) $f^{-1}(x) = \dfrac{1}{2x+1}$

   (b) $f^{-1}(x) = \dfrac{1}{2x} - 1$

   (c) $f^{-1}(x) = \dfrac{2x}{1}$

   (d) $f^{-1}(x) = \dfrac{1}{\frac{1}{2}x}$

   (e) $f^{-1}(x) = \dfrac{1}{2x}$

8. What is the domain of $f(g(x))$?

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 1) \cup (1, \infty)$

   (c) $(-\infty, 0) \cup (0, \infty)$

   (d) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

   (e) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

9. What is $f(g(x))$ (on its domain)?

   (a) $\dfrac{1}{2x} \cdot \dfrac{x-1}{x+1}$

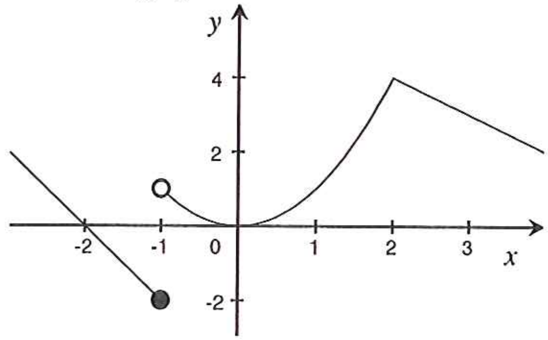
   (b) $\dfrac{x+1}{2x-2}$

   (c) $\dfrac{x}{2(x+1)} - 1$

   (d) $\dfrac{2x+2}{2x-2}$

   (e) $2x+1$

For this page, let $f(x)$ be the function graphed below:




10. Is the function continuous at $x = 2$ and why?

    (a) Yes, because the $\lim_{x\to 2} f(x) = f(2)$.

    (b) Yes, because $f$ is a continuous function.

    (c) No, because $\lim_{x\to 2^+} f(x) \neq f(2)$.

    (d) No, because 2 is a removable discontinuity.

    (e) None of the above.

11. Is the function differentiable at $x = -1$ and why?

    (a) Yes, because $\displaystyle\lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = f(-1)$.

    (b) Yes, because $f$ is differentiable at every point of its domain.

    (c) No, because $\displaystyle\lim_{h\to 0^-} \frac{f(-1+h) - f(-1)}{h} \neq \lim_{h\to 0^+} \frac{f(-1+h) - f(-1)}{h}$

    (d) No, because $\displaystyle\lim_{x\to -1} f(x)$ does not exist.

    (e) None of the above.

12. What is the range of $f^{-1}(x)$ if $f(x) = \dfrac{x+5}{2x-3}$?

(a) $(-\infty, 3) \cup (3, \infty)$
(b) $(-\infty, -1/2) \cup (-1/2, \infty)$
(c) $(-\infty, -5) \cup (-5, \infty)$
(d) $(-\infty, 3/2) \cup (3/2, \infty)$
(e) $(-\infty, -5) \cup (-5, 3/2) \cup (3/2, \infty)$

13. For what value of $a$ is $f(x)$ continuous at $x = 1$ if

$$f(x) = \begin{cases} 3 - ax & x < 1 \\ a + x & x \geq 1 \end{cases}$$

(a) $a = 0$
(b) $a = -1$
(c) $a = 1$
(d) $a = -2$
(e) none of the above

Calculate the derivatives of the following functions.

14. $f(x) = x^2 \sin(x)$

(a) $2x \cos(x)$

(b) $-2x \cos(x)$

(c) $2x \sin(x) + x^2 \cos(x)$

(d) $2x \sin(x) - x^2 \cos(x)$

(e) none of these

15. $f(x) = \tan(\sqrt{1 + x^{-1}})$

(a) $\sec^2\left(\dfrac{1}{2\sqrt{-\frac{1}{x^2}}}\right)$

(b) $\dfrac{1}{2\sqrt{-\frac{1}{x^2}}} \sec^2\left(\sqrt{1 + x^{-1}}\right)$

(c) $\dfrac{1}{2\sqrt{-\frac{1}{x^2}}} \sec^2\left(\sqrt{x}\right)$

(d) $-\dfrac{1}{x^2} \dfrac{1}{2\sqrt{1 + x^{-1}}} \sec^2\left(\sqrt{1 + x^{-1}}\right)$

(e) $-\dfrac{1}{x^2} \dfrac{1}{2\sqrt{x}} \sec^2(x)$

## Long answer questions

16. Let $f(x) = \dfrac{1}{x}$.

(a) State the limit definition of the derivative of $\dfrac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

(b) Calculate the derivative of $f(x)$ using the limit definition of the derivative. Show all of your work and explain your steps to receive any credit.

Since, as long as $h \neq 0$,

common denom.

cancel

$$\frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \frac{1}{h}\left(\frac{x-(x+h)}{(x+h)(x)}\right)$$

$$= \frac{1}{h}\left(-\frac{h}{(x+h)x}\right)$$

we have

cancels.

$$f'(x) = \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \lim_{h \to 0} -\frac{\cancel{h}}{\cancel{h}} \frac{1}{(x+h)x}$$

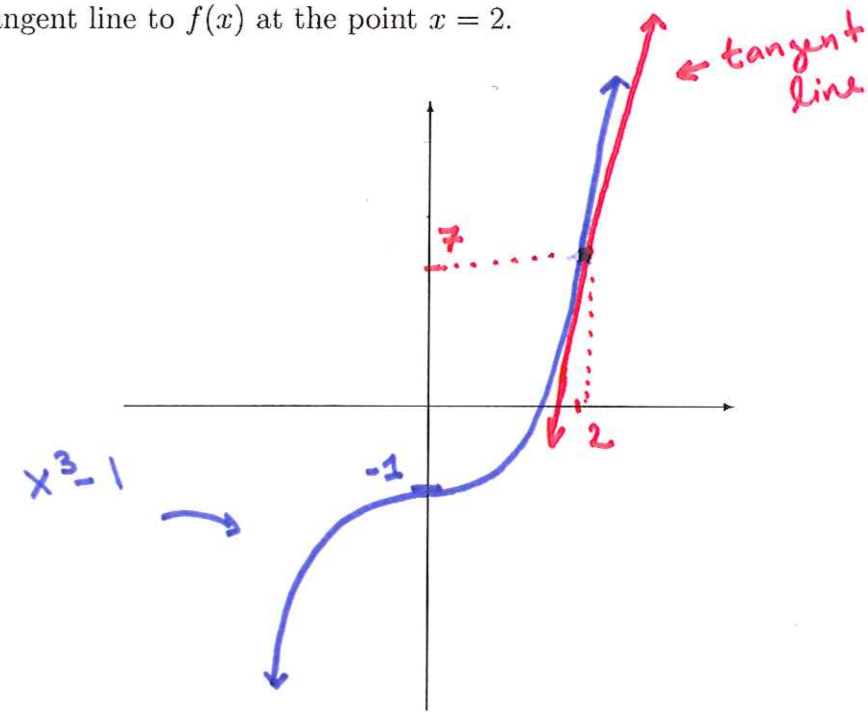
$$= -\frac{1}{x^2}.$$

17. Let $f(x) = x^3 - 1$.

(a) Give a rough sketch, on the same axes, of

i. $y = f(x)$, and

ii. the tangent line to $f(x)$ at the point $x = 2$.




(b) Give an equation for the line tangent to $f(x)$ at the point $x = 2$.

$\ell$ is tangent @ pt $(2, (2)^3 - 1)$

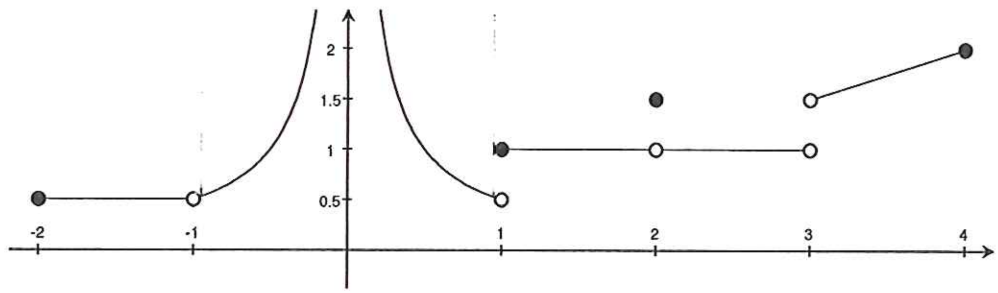
$= (2, 7)$

w/ slope

$$f'(2) = 3x^2\Big|_{x=2} = 12$$

$$\boxed{y - 7 = 12(x-2)}$$

18. Let $f(x)$ be the function graphed below.




(a) For what values of $x$ is the function $f(x)$ discontinuous on its domain?

$X = 1, 2$

(others are not in the domain)

(b) List the values where $f(x)$ has a removable discontinuity, and give the $y$-value to which we should redefine $f(x)$ at that point.

| removable discontinuity $x =$ | reassigned value $y =$ |
|---|---|
| 2 | 1 |

(c) List the values where $f(x)$ has a continuous extension, and give the $y$-value to which we should define $f(x)$ at that point.

| continuous extension at $x =$ | new value $y =$ |
|---|---|
| -1 | 0.5 |

notice,
this point had better
not be one of your answers
to part (a).