Hour Exam #1

Math 3

Oct. 19, 2011

Name (Print): ____
Last First

On this, the first of the two Math 3 hour-long exams in Fall 2010, and on the second hour-exam, and on the final examination I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:___________________________________

Instructor (circle):

Lahr (Sec. 1, 8:45) Crytser (Sec. 2, 11:15) Daugherty (Sec. 3, 12:30)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 15 multiple-choice problems worth 4 points each and 3 long-answer written problems worth a total of 40 points. Check to see that you have 11 pages of questions plus the cover page for a total of 12 pages.

Non-multiple choice questions:

Problem	Points	Score
16	10	
17	15	
18	15	
Total	40	

For this page, let $f(x) = \dfrac{3x+7}{x-2}$.

1. What is $\lim_{x\to\infty} f(x)$?

   (a) 0
   (b) 3
   (c) 1
   (d) ∞
   (e) The limit does not exist.

2. What is the domain of $f(x)$?

   (a) $(-\infty, \infty)$
   (b) $(-\infty, -1) \cup (-1, \infty)$
   (c) $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$
   (d) $(-\infty, 2) \cup (2, \infty)$
   (e) $(-2, 1) \cup (1, \infty)$

For this page, let $f(x) = \dfrac{2x}{x^4 - 5x^2 + 6}$.

3. What are the horizontal asymptotes of f?

   (a) $y = 2/5$

   (b) $y = 3$ and $y = 2$

   (c) $y = 0$

   (d) $x = \pm\sqrt{3}$

   (e) none of the above

4. What are the vertical asymptotes f?

   (a) $x = 0$

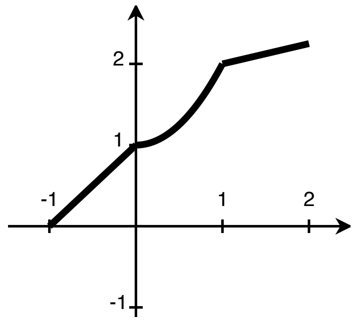
   (b) $x = \sqrt{2}$

   (c) $x = \pm\sqrt{2}$ and $x = \pm\sqrt{3}$

   (d) $y = 2$ and $y = 3$

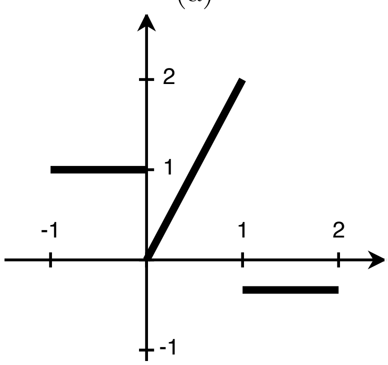
   (e) none of the above

Suppose the graph of the function $f(x)$ looks like



5. Of the following graphs, which could be the graph of $f'(x)$?

(a)



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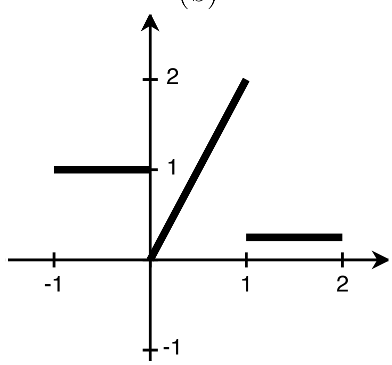

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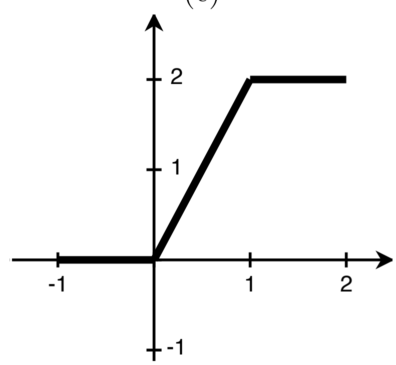

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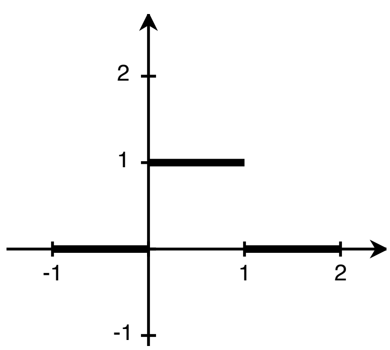
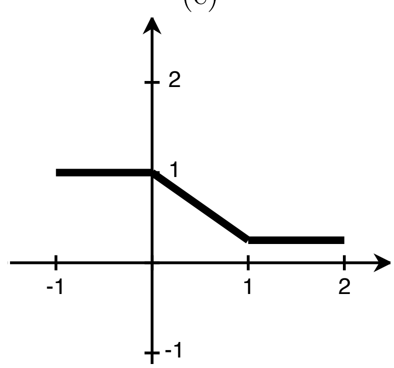
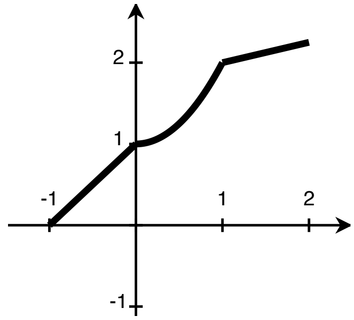


(e)

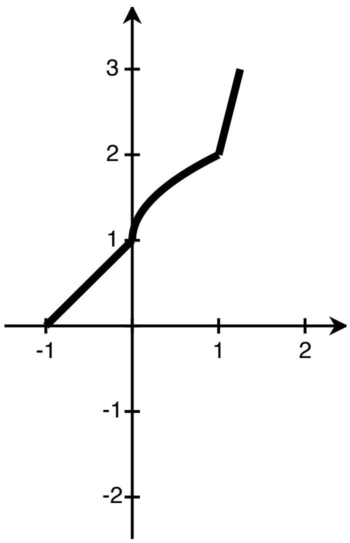


Again, suppose the graph of the function $f(x)$ looks like

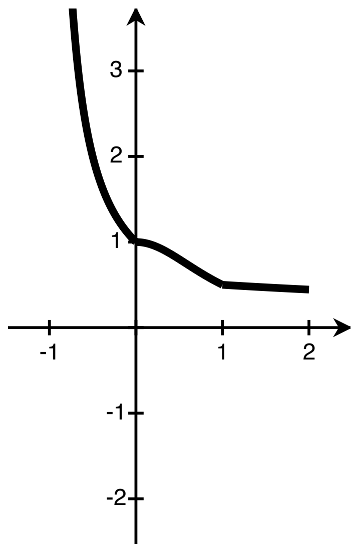


6. Of the following graphs, which is the graph of $f^{-1}(x)$?

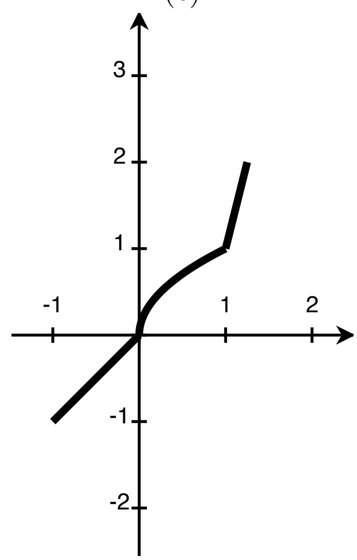
(a)



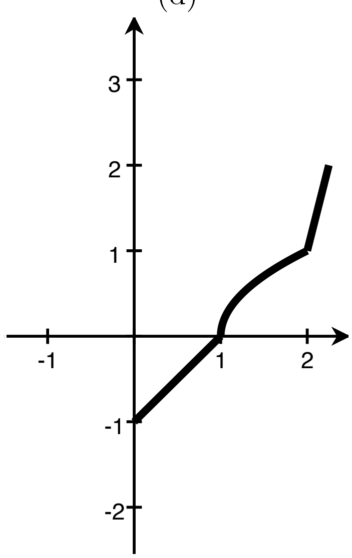
(b)



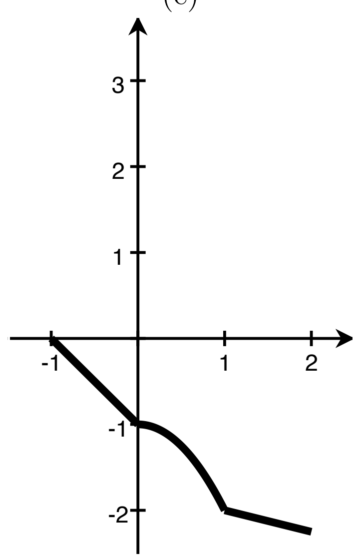
(c)



(d)



(e)



For this page, let $f(x) = \dfrac{1}{2x}$ and $g(x) = \dfrac{x-1}{x+1}$.

7. What is $f^{-1}(x)$?

   (a) $f^{-1}(x) = \dfrac{1}{2x+1}$

   (b) $f^{-1}(x) = \dfrac{1}{2x} - 1$

   (c) $f^{-1}(x) = \dfrac{2x}{1}$

   (d) $f^{-1}(x) = \dfrac{1}{\frac{1}{2}x}$

   (e) $f^{-1}(x) = \dfrac{1}{2x}$

8. What is the domain of $f(g(x))$?

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 1) \cup (1, \infty)$

   (c) $(-\infty, 0) \cup (0, \infty)$

   (d) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

   (e) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

9. What is $f(g(x))$ (on its domain)?

   (a) $\dfrac{1}{2x} \cdot \dfrac{x-1}{x+1}$

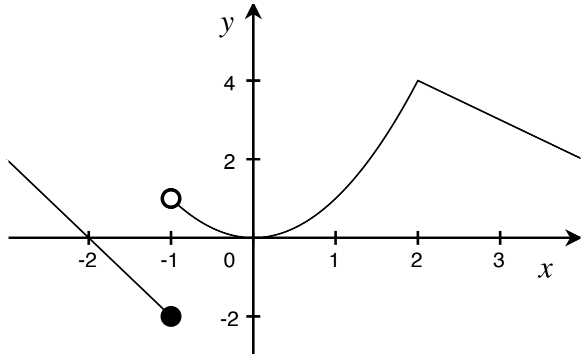
   (b) $\dfrac{x+1}{2x-2}$

   (c) $\dfrac{x}{2(x+1)} - 1$

   (d) $\dfrac{2x+2}{2x-2}$

   (e) $2x + 1$

For this page, let $f(x)$ be the function graphed below:



10. Is the function continuous at $x = 2$ and why?

   (a) Yes, because the $\lim_{x\to 2} f(x) = f(2)$.

   (b) Yes, because f is a continuous function.

   (c) No, because $\lim_{x\to 2^+} f(x) \neq f(2)$.

   (d) No, because 2 is a removable discontinuity.

   (e) None of the above.

11. Is the function differentiable at $x = -1$ and why?

   (a) Yes, because $\displaystyle\lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = f(-1)$.

   (b) Yes, because f is differentiable at every point of its domain.

   (c) No, because $\displaystyle\lim_{h\to 0^-} \frac{f(-1+h) - f(-1)}{h} \neq \lim_{h\to 0^+} \frac{f(-1+h) - f(-1)}{h}$

   (d) No, because $\displaystyle\lim_{x\to -1} f(x)$ does not exist.

   (e) None of the above.

12. What is the range of $f^{-1}(x)$ if $f(x) = \dfrac{x+5}{2x-3}$?

(a) $(-\infty, 3) \cup (3, \infty)$

(b) $(-\infty, -1/2) \cup (-1/2, \infty)$

(c) $(-\infty, -5) \cup (-5, \infty)$

(d) $(-\infty, 3/2) \cup (3/2, \infty)$

(e) $(-\infty, -5) \cup (-5, 3/2) \cup (3/2, \infty)$

13. For what value of a is $f(x)$ continuous at $x = 1$ if

$$f(x) = \begin{cases} 3 - ax & x < 1 \\ a + x & x \geq 1 \end{cases}$$

(a) $a = 0$

(b) $a = -1$

(c) $a = 1$

(d) $a = -2$

(e) none of the above

Calculate the derivatives of the following functions.

14. $f(x) = x^2 \sin(x)$

    (a) $2x\cos(x)$

    (b) $-2x\cos(x)$

    (c) $2x\sin(x) + x^2\cos(x)$

    (d) $2x\sin(x) - x^2\cos(x)$

    (e) none of these

15. $f(x) = \tan(\sqrt{1 + x^{-1}})$

    (a) $\sec^2\left(\dfrac{1}{2\sqrt{-\frac{1}{x^2}}}\right)$

    (b) $\dfrac{1}{2\sqrt{-\frac{1}{x^2}}}\sec^2\left(\sqrt{1+x^{-1}}\right)$

    (c) $\dfrac{1}{2\sqrt{-\frac{1}{x^2}}}\sec^2\left(\sqrt{x}\right)$

    (d) $-\dfrac{1}{x^2}\dfrac{1}{2\sqrt{1+x^{-1}}}\sec^2\left(\sqrt{1+x^{-1}}\right)$

    (e) $-\dfrac{1}{x^2}\dfrac{1}{2\sqrt{x}}\sec^2\left(x\right)$

Long answer questions

16. Let $f(x) = \dfrac{1}{x}$.

    (a) State the limit definition of the derivative of $\dfrac{1}{x}$.

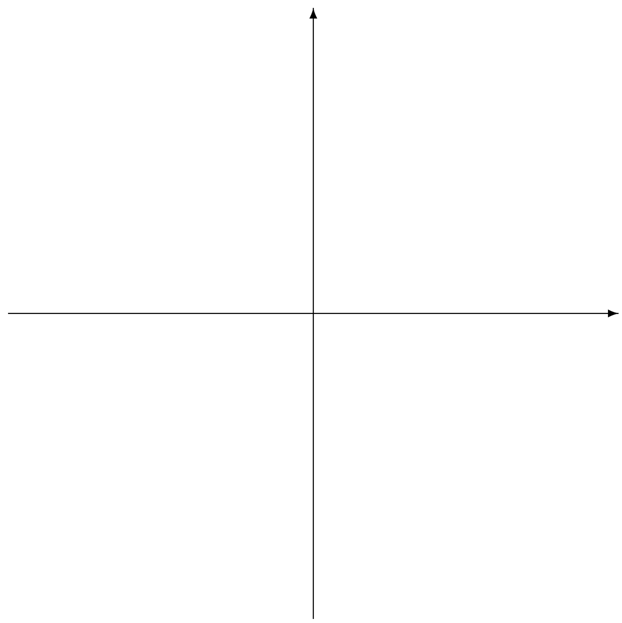
    (b) Calculate the derivative of $f(x)$ using the limit definition of the derivative. Show all of your work and explain your steps to receive any credit.

17. Let $f(x) = x^3 - 1$.

(a) Give a rough sketch, on the same axes, of

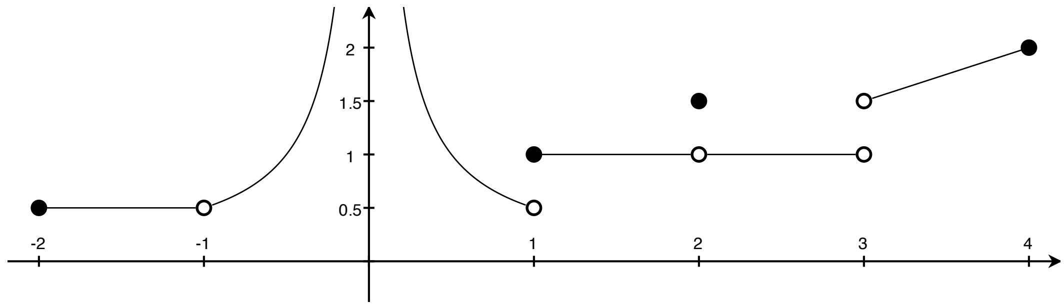
i. $y = f(x)$, and

ii. the tangent line to $f(x)$ at the point $x = 2$.



(b) Give an equation for the line tangent to $f(x)$ at the point $x = 2$.

18. Let $f(x)$ be the function graphed below.



(a) For what values of x is the function $f(x)$ discontinuous on its domain?

(b) List the values where $f(x)$ has a removable discontinuity, and give the y-value to which we should redefine $f(x)$ at that point.

removable discontinuity $x =$	reassigned value $y =$

(c) List the values where $f(x)$ has a continuous extension, and give the y-value to which we should define $f(x)$ at that point.

continuous extension at $x =$	new value $y =$