Patterns in Time Series and Dynamical Systems

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Abstract

Recently-developed methods for estimating the entropy of time series are based on the short-term behavior of an underlying system. We encode this short-term behavior using permutations, called (ordinal) patterns. Motivated by the utility of permutation-based techniques and the elegant connections between the permutation structure of periodic points and topological entropy, we study the relationship between ordinal patterns and dynamical systems.

In this thesis, we show how patterns realized by finite orbits can be interpreted in the setting of classical combinatorial dynamics. We also present a dynamical interpretation of a combinatorial bijection relating the cycle structure and the one-line notation of permutations. Next, we obtain a concise characterization of the allowed patterns of signed-sawtooth maps, which is a class of functions that generalize the shift and tent maps. Specifically, for the negative shift, equivalent to $M_{-k}(x) = -kx \bmod 1$, we give a formula for the number of distinct allowed patterns. Additionally, for negative real shifts, called $-\beta$-shifts, we show that the set of allowed patterns increases with β in the sense of containment. We then express the infimum value β for which a given pattern appears as the largest real root of an associated polynomial.

Finally, we extend the interpretation of permutation entropy as a divergence from white noise to give a measure of inefficiency for stock market data. Particularly, we introduce a null model motivated by economic theory and show that this perspective gives an interesting measure of complexity in this setting.