On the Poisson Relation for Lens Spaces

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Abstract

Motivated by quantum mechanics and geometric optics, it is a long-standing problem whether the length spectrum of a compact Riemannian manifold can be recovered from its Laplace spectrum. One route to proving that the length spectrum depends on the Laplace spectrum is by computing the singular support of the trace of the corresponding wave group. Indeed, it is well-known that the singular support of the trace of the wave group is a subset of the periods of the geodesic flow: a relation known as the Poisson relation. Proving that the Poisson relation is an equality for a given manifold yields an effective means of recovering the length spectrum of a space from its Laplace spectrum. We investigate whether the Poisson relation is an equality on lens spaces. We show that all lens spaces satisfy the clean intersection hypothesis of Duistermaat and Guillemin. Moreover, we show that the Poisson relation is an equality for homogeneous lens spaces. Finally, we explicitly compute the highest order wave invariants to show that on 3-dimensional lens spaces with fundamental group of prime order the Poisson relation is an equality. Besides spheres and real projective spaces, these are the only other examples of spherical space forms on which it is known that the Poisson relation is an equality.