# Equivalences on Patterns in Random Walks

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*Abstract*

In the past decade, the use of ordinal patterns in the analysis of time series and dynamical systems has become an important tool. Ordinal patterns (otherwise known as a permutation patterns) are found in time series by taking $n$ data points at evenly-spaced time intervals and mapping them to a length $n$ permutation determined by relative ordering. The frequency with which certain patterns occur is a useful statistic for such series. However, the behavior of the frequency of pattern occurrence is unstudied for most models. We look at the frequency of pattern occurrence in random walks in discrete time and define a natural equivalence relation on permutations under which equivalent patterns appear with equal frequency, regardless of probability distribution. Toward the goal of characterizing the corresponding equivalence classes for this relation, we introduce a diagrammatic interpretation of permutations and develop a surrounding theory. Through a partitioning of these diagrams, we provide a complete characterization of the equivalence classes and describe a method for efficiently computing the members of a class. We then conjecture that the classes describe all patterns that appear with equal frequency, regardless of probability distribution; we provide a characterization of permutations that are related by frequency as progress toward proving this conjecture.