Roto-Translation Space and the Visual Cortex

Gregory C. Petrics

Abstract

We present two techniques for producing solutions to the minimal surface problem with Dirichlet boundary conditions in the roto-translation space realized as a sub-Riemannian manifold. One technique improves upon a level set method introduced by G. Citti and A. Sarti. The second technique treats surfaces as graphs. The graph approach is more computationally efficient, but loses some of the topological advantages of the level set approach. Both techniques produce numerical solutions of the minimal surface problem by evolving surfaces according to mean curvature. These are used as candidates for a theoretical investigation into minimal surfaces. These surfaces are applied to a problem from neuroscience by using the model of the primary visual cortex by the roto-translation space of G. Citti and A. Sarti in which minimal spanning surfaces are used to provide image completions for incomplete visual data. We also study a discrete formulation of the minimal surface problem in the roto-translation space, produce examples of discrete solutions to this formulation, and discuss the meaning of these solutions in the visual cortex. We then study the class of surfaces in the roto-translation space which are C^2, sub-Riemannian minimal, and almost everywhere locally graphs over $\mathbb{R}^2$. These surfaces are analyzed by interfacing the sub-Riemannian geometry of the roto-translation space with a study of functions which are differentiable almost everywhere on the plane. We list and compare several examples, and prove that an image of concentric circles occluded by a disc centered on the origin has an infilling using a more complex image we call the catseye. This expands the literature on global sub-Riemannian minimal surface theory into non-Carnot groups. Finally we use the model of the visual cortex by the roto-translation space to study the induced grating optical illusion. We produce a family of image completions of a sinewave image that respect the roto-translation space model. The existence of multiple image completions is suggested as a possible explanation of the induced grating effect.