

# Firefighting on Geometric Graphs

Amir Barghi

## *Abstract*

In the Firefighter Problem, a fire starts at a vertex of a graph, and in discrete time units, it spreads from burned vertices to their neighbors unless they are protected by one of the  $f$  firefighters that are deployed every turn. Once burned or protected, a vertex remains in that state. In the case of finite graphs, the firefighters wish to minimize the damage or the time that the fire rages. When a fire starts in an infinite graph, the key question is whether the fire can be stopped. In this thesis, we will consider three different models for how the vertices of an infinite graph are distributed on the Euclidean plane. All these models have a geometric aspect: in the first two models, random and extremal geometric graphs, the geometric aspect is due to the adjacency criterion, and in the third model, plane-like graphs, it is due to the specific representation of an infinite planar graph on the Euclidean plane. Our objective is to study the conditions under which the fire can be contained. In particular, our results focus mostly on upper and lower bounds on the number of firefighters needed to prevent a fire from spreading indefinitely.