Ph.D. Thesis

Martin Malandro
Advisor: Dan Rockmore

Fast Fourier Transforms for Inverse Semigroups

Abstract

In this thesis, we develop a theory of Fourier analysis and fast Fourier transforms (FFTs) for finite inverse semigroups. Our results generalize results in the theory of Fourier analysis for finite groups. There is a general method for generating the irreducible representations of an inverse semigroup, and we use this method to prove that the problem of creating FFTs for inverse semigroups can be reduced to the problems of creating FFTs for their maximal subgroups and creating fast zeta transforms for their poset structures. We then use this result to create FFTs for certain inverse semigroups of interest n in particular, for the rook monoid and its wreath products by arbitrary finite groups. We prove a number of results that are important in the theory of Fourier analysis for inverse semigroups along the way. Finally, we use these results to provide an application to the statistical analysis of partially ranked data. Generally speaking, our tools include elements from group and semigroup representation theory, the theory of partially ordered sets and Möbius inversion, and the theory of noncommutative rings.