Ph. D. Thesis

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Results on off-branch numbers

I consider the problem of extending certain invariants of subsets of natural numbers to their equivalents for subsets of larger cardinals. The two specific questions I will address are the extension of the off-branch number $\mathfrak{o}$ and its relation to other invariants; and the effect of replacing the ideal of sets of size $< \kappa$ with the ideal of non-stationary sets in cardinal invariants on κ, with particular attention to the splitting number $\mathfrak{s}(\kappa)$.

The *off-branch number* on κ is $\mathfrak{o}(\kappa)$, the least number of off-branch subsets of $2^{<\kappa}$ which together with the branches of 2^κ form a mad family on $2^{<\kappa}$. The ZFC-provable inequalities I show are that $\mathfrak{a}(\kappa) \leq \mathfrak{o}(\kappa)$ and $\mathfrak{non}(\mathcal{M})(\kappa) \leq \mathfrak{o}(\kappa)$ for κ inaccessible. The consistency results I find are that $\mathfrak{a}(\kappa) < \mathfrak{o}(\kappa)$ and $\mathfrak{non}(\mathcal{M})(\kappa) < \mathfrak{o}(\kappa)$ are possible if κ is inaccessible, and that $\mathfrak{o}(\kappa) < 2^\kappa$ is possible if κ is indestructibly weakly compact.

For an ideal $\mathcal{I}$ on κ, the *$\mathcal{I}$-splitting number* is $\mathfrak{s}_\mathcal{I}(\kappa)$, the least size of a family $\mathcal{S}$ of sets in $\mathcal{I}^+$ such that for every set $X \in \mathcal{I}^+$ there is a set $S \in \mathcal{S}$ with $X \cap S, X - S \in \mathcal{I}^+$. For $\mathcal{I}$ the ideal of sets of size $< \kappa$, this is the usual splitting number of κ, whose being large has been shown to be equivalent to large cardinal properties; I obtain similar results for $\mathcal{I} = NS(\kappa)$, the ideal of non-stationary subsets of κ.