Math 101: Linear and multilinear algebra

ORC syllabus

This course treats the subject of linear and multilinear algebra from an abstract point of view. Topics include bilinear forms, tensor products, algebras over a field, symmetric and exterior powers, and universal properties. Applications include the representation and character theory of finite groups.

References.

[DF] David Dummit and Richard Foote, *Abstract algebra*, 3rd ed., John Wiley and Sons, 2004.

[FIS] Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence, *Linear algebra*, 4th ed., Pearson, 2002.

[R] Steven Roman, *Advanced linear algebra*, 3rd ed., Springer, 2008.

Linear and multilinear algebra.

1. *Coordinate-free linear algebra review.* Universal property of linear independence and basis. Endomorphism algebra and the matrix algebra. Quotient spaces and the isomorphism theorem. Direct sums. [R, 3–4]

2. *Bilinear forms.* Gram matrix. Symmetric, skew-symmetric, alternating bilinear forms. Orthogonal complements. Isometries. [R, 11]

3. *Inner product spaces.* Orthogonal and orthonormal bases. Gram–Schmidt process. Adjoint. Normal and self-adjoint operators. Orthogonal and unitary groups. The spectral theorem. Singular value decomposition. [R, 12; FIS, 6.1–6.7]

4. *Multilinear algebra: Dual spaces.* Dual basis. Dual map. Evaluation map. Bilinear forms and dual spaces. [R, 3]

5. *Multilinear algebra: Tensor product.* Tensor product via universal property and free vector space quotient. Tensor product basis. Space of bilinear forms. [R, 14]

6. *Algebras over a field.* Examples from field theory, polynomial rings, matrix algebras, quaternions. Free associative algebras, tensor algebra, gradings. [R, 18, 14]

7. *Multilinear algebra: Symmetric powers.* Symmetric square and symmetric bilinear forms. Symmetric algebra and universal property. [R, 14]

8. *Multilinear algebra: Exterior powers.* Exterior square and alternating bilinear forms. Exterior algebra. Universal property. Exterior powers, minors, determinants. [R, 14]

Representation theory.

1. Linear actions and modules over group rings. Reducible, indecomposable, semisimple modules. Maschke's theorem. [DF, 18.1]
2. Wedderburn's theorem. Schur's lemma. Decomposition of the group ring. [DF, 18.2]
3. Character theory and orthogonality relations and character tables. [DF, 18.3, 19.1]